C.I.S. 1.5 (Science Section)
Brooklyn College
Professor Langsam

Assignment #1 – Centripetal Force[1]

Newton's first law of motion was stated as "A body continues to move at a constant velocity unless acted upon by a force." Remembering that "constant velocity" means traveling at constant speed in a straight line, we realize that Newton knew that circular motion is not natural. A force is required to maintain circular motion. When a car turns a corner this force is provided by the friction between the tires and the road. When a ball is twirled at the end of a string this force is provided by the tension on the string. Similarly, roller coaster cars are forced through a loop by the track applying a centripetal force on them. The reactive centrifugal force of the cars, associated with their inertia, holds them on the track. And as the earth orbits around the sun this force is provide by the gravitational attraction between the two bodies.

Centripetal force is a force that makes a body follow a curved, as opposed to straight, path; it is always directed orthogonal to the velocity of the body, toward the instantaneous center of curvature of the path. The term centripetal force comes from the Latin words centrum ("center") and petere ("tend towards", "aim at"), signifying that the force is directed inward toward the center of curvature of the path. Isaac Newton's description was: "A centripetal force is that by which bodies are drawn or impelled or in any way tend, towards a point as to a center." Any force (gravitational, electromagnetic, etc.) or combination of forces can act to provide a centripetal force. An example for the case of uniform circular motion is shown in Figure 1.

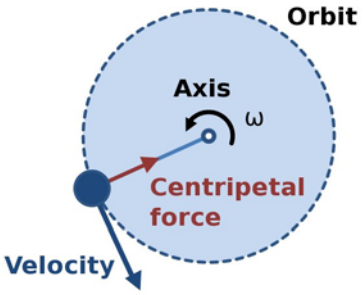

Figure 1: A simple example corresponding to uniform circular motion. A ball is tethered to a rotational axis and is rotating counterclockwise around the specified path at a constant angular rate ω. The velocity of the ball is a vector tangential to the orbit, and is continuously changing direction, a change requiring a radially inward directed centripetal force. The centripetal force is provided by the tether, which is in a state of tension.

Centripetal force is defined by:

$$\text{Centripetal Force} = F_{cent} = \frac{mv_t^2}{r}$$

[1] www.wikipedia.com

where:

F_{cent} = centripetal force (in Newtons)

m = mass (in *kg*)

v_t = tangential velocity in (*m/sec*)(see Figure 1)

r = radius of curvature (in *m*)

Since the velocity of an object changes when it moves in a circle we must have an acceleration. An acceleration requires an unbalanced force, which in this case is the centripetal force. We use Newton's second law ($F = ma$) to define this ***centripetal acceleration***.

$$\text{Centripetal Force} = F_{cent} = ma_{cent} = \frac{mv^2}{r}$$

and, therefore

$$\text{Centripetal acceleration} = a_{cent} = \frac{v_t^2}{r}$$

Note: Even when an object travels in a circle with constant tangential speed, the object experiences an unbalanced force (the centripetal force) and it is accelerating (the centripetal acceleration).

Example: A 1200 kg (2,640 lb) car is turning a corner at a speed of 8 m/sec (18 mph), and it travels along the arc of a circle in the process. If the radius of this circle is 9 m, what is the centripetal force required to hold the car in the circular path? What is the car's centripetal acceleration?

$$F_{cent} = \frac{mv_t^2}{r} = \frac{(1200kg)(8m/\sec)^2}{9meters} = 8530N$$

and

$$a_{cent} = \frac{v_t^2}{r} = \frac{(8m/\sec)^2}{9m} = 7.1m/\sec^2$$

This force must be supplied by the friction force of the pavement on the tires. If the pavement is wet or icy so that there is little friction between the tires and the road, the friction force on the

tires will perhaps not be large. In that event the car will skid out of a circular path (into more nearly a straight line) and may not be able to negotiate the curve.

Write a C++ program that calculates the centripetal force (F_{cent}) and centripetal acceleration (a_{cent}) of objects having the following masses and travelling with tangential velocities (v_t) about a curve of radius (r).

For each set of data your program should print a message such as:

FOR AN OBJECT OF MASS _____ KG, AND A TANGENTIAL VELOCITY OF _____ METERS PER SECOND TRAVELING ABOUT A CURVE OF RADIUS_____ METERS, THE CENTRIPETAL FORCE IS _____ NEWTONS AND THE CENTRIPETAL ACCELERATION IS _____ METERS PER SECOND SQUARED.

Calculate the above for the following sets of data:

Mass (kg)	**Tangential Velocity (meters/second)**	**Radius (meters)**
1200	8	9
1	1	1
1	0.5	1
.5	1	1
100	5	10
5	10	5

Use the trailer method to signal the end of the data. Be sure to use meaningful variables, comment your program, and use a consistent style.